

Lecture 8: Nonclassical light

- Squeezing
- Photon anti-bunching

Squeezing: The examples we have given so far for nonclassical light have been rather intuitive, and so has been the notion of nonclassicality. Here, we will seek for properly defined measures of nonclassicality that can be applied to arbitrary quantum states, and which unambiguously can discriminate between quantum states that do have a classical counterpart and those that do not. In particular, we focus on two criteria for nonclassicality, squeezing and photon anti-bunching.

We have seen in an earlier lecture that there are quantum states of light that show electric-field fluctuations below the vacuum level given by Eq. (4.17), $\langle 0 | [\Delta \hat{E}_k(\mathbf{r})]^2 | 0 \rangle = \omega^2 |A_k(\mathbf{r})|^2$. Hence, the definition of squeezing can be given in the following way. A quantum state is said to be squeezed if the electric-field fluctuations in that state are, for some phase values, smaller than the vacuum fluctuations,

$$\langle [\Delta \hat{E}_k(\mathbf{r})]^2 \rangle < \langle 0 | [\Delta \hat{E}_k(\mathbf{r})]^2 | 0 \rangle. \quad (8.1)$$

We can rewrite this condition in the following way:

$$\begin{aligned} \langle [\Delta \hat{E}_k(\mathbf{r})]^2 \rangle &= \langle \hat{E}_k^2(\mathbf{r}) \rangle - \langle \hat{E}_k(\mathbf{r}) \rangle^2 = \langle [\hat{E}_k^{(+)}(\mathbf{r}) + \hat{E}_k^{(-)}(\mathbf{r})]^2 \rangle - \langle \hat{E}_k(\mathbf{r}) \rangle^2 \\ &= \langle : [\Delta \hat{E}_k(\mathbf{r})]^2 : \rangle + \langle [\hat{E}_k^{(+)}(\mathbf{r}), \hat{E}_k^{(-)}(\mathbf{r})] \rangle \end{aligned} \quad (8.2)$$

where the symbol $: \hat{O} :$ means that the operator \hat{O} has been put into normal order with respect to the creation and annihilation operators. Then it is easy to see that $\langle [\hat{E}_k^{(+)}(\mathbf{r}), \hat{E}_k^{(-)}(\mathbf{r})] \rangle = \omega^2 |A_k(\mathbf{r})|^2 \langle [\hat{a}, \hat{a}^\dagger] \rangle$ represents just the vacuum fluctuations. Applying this relation to Eq. (8.1), we obtain

$$\boxed{\langle : [\Delta \hat{E}_k(\mathbf{r})]^2 : \rangle < 0} \quad (8.3)$$

as our condition for squeezing.

We have yet to show that a quantum state fulfilling (8.3) has no classical counterpart. Remember that, in classical statistics, we can always find a classical probability distribution $p_{\text{cl}}(\alpha)$, i.e. a well-defined non-negative function, from which we can determine the

expectation value of a classical variable O ,

$$\langle O \rangle = \int d^2\alpha O(\alpha, \alpha^*) p_{\text{cl}}(\alpha). \quad (8.4)$$

With regard to the squeezing condition (8.3), we have to associate the moments $O(\alpha, \alpha^*)$ in Eq. (8.4) with the variance of the electric-field strength, $[\Delta E_k(\mathbf{r}, \alpha, \alpha^*)]^2$, which itself is a non-negative function. Hence, for the inequality (8.3) to hold, the probability distribution associated with this variance must take negative values somewhere in phase space which violates our assumption of non-negativity. Therefore, states that obey condition (8.3) cannot have a classical counterpart. This holds particularly for the class of Gaussian states that we have already called squeezed states. However, there are many more nonclassical states obeying this squeezing condition.

Photon anti-bunching: Another nonclassicality condition is obtained by studying the second-order (normally and time-ordered) intensity correlation function

$$\begin{aligned} G^{(2)}(\mathbf{r}_1, t + \tau, \mathbf{r}_2, t) &= \langle : \hat{I}(\mathbf{r}_1, t + \tau) \hat{I}(\mathbf{r}_2, t) : \rangle \\ &= \langle \hat{E}_i^{(-)}(\mathbf{r}_2, t) \hat{E}_j^{(-)}(\mathbf{r}_1, t + \tau) \hat{E}_j^{(+)}(\mathbf{r}_1, t + \tau) \hat{E}_i^{(+)}(\mathbf{r}_2, t) \rangle. \end{aligned} \quad (8.5)$$

Such a correlation function can be measured by a photocounting-correlation measurement in a Hanbury Brown–Twiss set-up (see Figure 9).

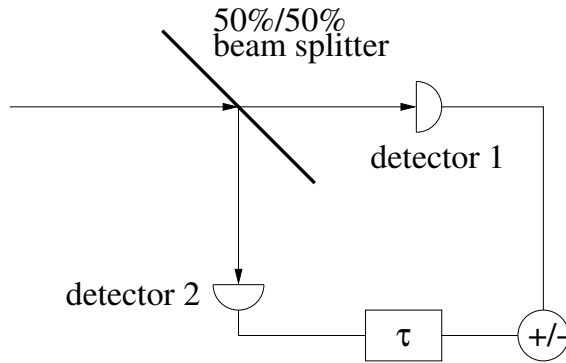


FIG. 9: Scheme of an Hanbury Brown–Twiss experiment for measuring intensity correlations. A signal is split at a symmetric beam splitter and recorded at two photodetectors. One measurement signal is delayed by a variable time τ .

The classical analogue to Eq. (8.5) would be

$$G_{\text{cl}}^{(2)}(\mathbf{r}_1, t + \tau, \mathbf{r}_2, t) = \langle I(\mathbf{r}_1, t + \tau) I(\mathbf{r}_2, t) \rangle \quad (8.6)$$

where $I(\mathbf{r}, t) = \mathbf{E}^{(-)}(\mathbf{r}, t) \cdot \mathbf{E}^{(+)}(\mathbf{r}, t)$ is the classical intensity. With the help of the joint probability distribution function $p_{\text{cl}}(I, t + \tau, I', t)$, this can be written as

$$G_{\text{cl}}^{(2)}(\mathbf{r}_1, t + \tau, \mathbf{r}_2, t) = \int dI dI' p_{\text{cl}}(I, t + \tau, I', t) I(\mathbf{r}_1) I'(\mathbf{r}_2). \quad (8.7)$$

Applying the Cauchy–Schwarz inequality to the intensity correlation function (8.6), we find that

$$G_{\text{cl}}^{(2)}(\mathbf{r}_1, t + \tau, \mathbf{r}_2, t) \leq [\langle I^2(\mathbf{r}_1, t + \tau) \rangle]^{1/2} [\langle I^2(\mathbf{r}_2, t) \rangle]^{1/2} \quad (8.8)$$

which implies that

$$G_{\text{cl}}^{(2)}(\mathbf{r}_1, t + \tau, \mathbf{r}_2, t) \leq \left[\int dI p_{\text{cl}}(I, t + \tau) [I(\mathbf{r}_1)]^2 \right]^{1/2} \left[\int dI' p_{\text{cl}}(I', t) [I'(\mathbf{r}_2)]^2 \right]^{1/2}. \quad (8.9)$$

Here, the marginal probability distribution $p_{\text{cl}}(I, t) = \int dI' p_{\text{cl}}(I, t, I', t)$ has been used. In the steady-state regime, $t \rightarrow \infty$, we define

$$G^{(2)}(\tau) = \lim_{t \rightarrow \infty} G_{\text{cl}}^{(2)}(\mathbf{r}_1, t + \tau, \mathbf{r}_2, t), \quad (8.10)$$

so that the Cauchy–Schwarz inequality (8.9) takes the form

$$G^{(2)}(\tau) \leq G^{(2)}(0). \quad (8.11)$$

This means that classical statistics predicts a non-increasing initial slope for the steady-state intensity correlation function $G^{(2)}(\tau)$. Hence, in a Hanbury Brown–Twiss experiment the probability of finding coincidence counts at zero time delay τ is larger than for non-zero time delay. Thus, in classical optics, photons (if one can speak about photons there) tend to arrive together rather than individually. This phenomenon is called *photon bunching*. On the other hand, a quantum state that satisfies

$$\boxed{G^{(2)}(\tau) > G^{(2)}(0)} \quad (8.12)$$

cannot have a classical counterpart. Here, photons tend to arrive separate from each other rather than in lumps. This quantum phenomenon is thus called *photon anti-bunching*.

Second-order coherence: As a measure for the amount of anti-bunching, let us define the normalized intensity correlation function, or simply second-order coherence, as

$$g^{(2)}(\tau) = \lim_{t \rightarrow \infty} \frac{\langle : \hat{I}(\mathbf{r}_1, t + \tau) \hat{I}(\mathbf{r}_2, t) : \rangle}{\langle \hat{I}(\mathbf{r}_1, t + \tau) \rangle \langle \hat{I}(\mathbf{r}_2, t) \rangle}. \quad (8.13)$$

The function $g^{(2)}(\tau)$ measures correlations of photodetection events at the two detectors in the Hanbury Brown–Twiss experiment that are separated by a delay time τ . For zero time delay, $g^{(2)}(0)$ is related to the normalized intensity variance. If the photodetection probabilities are independent, then $g^{(2)}(0) = 1$. This is the case for a coherent state for which $\hat{\mathbf{E}}^{(+)}(\mathbf{r}_i)|\alpha\rangle = i\omega\mathbf{A}(\mathbf{r}_i)\alpha|\alpha\rangle$, and hence $\langle\alpha|\hat{I}(\mathbf{r}_1)\hat{I}(\mathbf{r}_2)|\alpha\rangle = \langle\alpha|\hat{I}(\mathbf{r}_1)|\alpha\rangle\langle\alpha|\hat{I}(\mathbf{r}_2)|\alpha\rangle$ (see Fig. 10).

The situation is changed if we look at light prepared in a single-photon Fock state $|1\rangle$. Here we find that $\hat{\mathbf{E}}^{(+)}(\mathbf{r}_i)|1\rangle = i\omega\mathbf{A}(\mathbf{r}_i)|0\rangle$ and therefore $\hat{\mathbf{E}}^{(+)}(\mathbf{r}_1)\hat{\mathbf{E}}^{(+)}(\mathbf{r}_2)|1\rangle = 0$. This immediately implies that $g^{(2)}(0) = 0$ (see Fig. 10). The photodetection events are correlated in such a way that there are no simultaneous events at all. This is rather intuitive because there is one and only one photon present at any one time. Once that photon has been registered in one of the two detectors, it is destroyed and cannot appear at the other detector. A stream of light pulses each prepared in single-photon Fock states arrive time-separated and are thus anti-bunched.

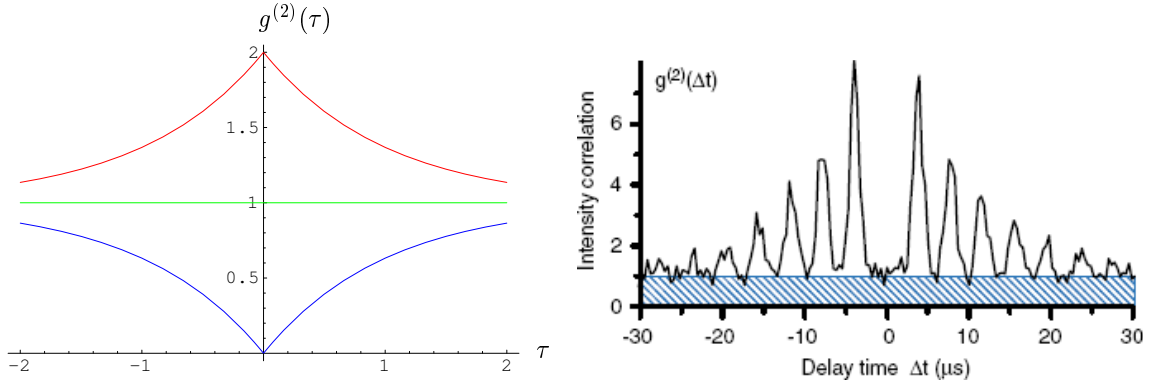


FIG. 10: Second-order intensity correlation function $g^{(2)}(\tau)$ (left figure) for light prepared in a single-photon Fock state (blue), coherent state (green) and for thermal light (red). The width of the distribution is a measure for the temporal coherence length. The panel on the right shows a typical experimental measurement result for pulsed single-photon sources.

Roy J. Glauber received the Nobel Prize 2005 ”for his contribution to the quantum theory of optical coherence”.